CS 188: Artificial Intelligence
Spring 2010

Lecture 23: Perceptrons
4/15/2010

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Many slides adapted from Dan Klein.

Announcements

- Project 4: due tonight.
- W7: out tonight.
- Final Contest: up and running!

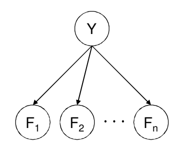
Outline

- Naïve Bayes recap
- Smoothing
- Generative vs. Discriminative
- Perceptron

Recap: General Naïve Bayes

- A general *naïve Bayes* model:
  - Y: label to be predicted
  - $\mathrm{F}_1, \ldots, \mathrm{F}_n$: features of each instance

$$P(\mathrm{Y}, \mathrm{F}_1 \ldots \mathrm{F}_n) = P(\mathrm{Y}) \prod_i P(\mathrm{F}_i|\mathrm{Y})$$



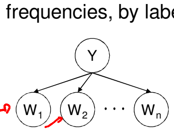
Naïve Bayes Training

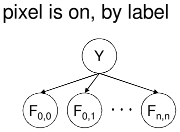
- Data: labeled instances, e.g. emails marked as spam/ham by a person
  - Divide into training, held-out, and test
- Features are known for every training, held-out and test instance
- Estimation: count feature values in the training set and normalize to get maximum likelihood estimates of probabilities
- Smoothing (aka regularization): adjust estimates to account for unseen data

Example Naïve Bayes Models

- Bag-of-words for text
  - One feature for every word position in the document
  - All features share the same conditional distributions
  - Maximum likelihood estimates: word frequencies, by label
- Pixels for images
  - One feature for every pixel, indicating whether it is on (black)
  - Each pixel has a different conditional distribution
  - Maximum likelihood estimates: how often a pixel is on, by label

P(W|Y)





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Recap: Laplace Smoothing

- Laplace's estimate (extended):
  - Pretend you saw every outcome k extra times

$$P_{LAP,k}(x) = \frac{c(x)+k}{c(\cdot)+k|X|}$$

  - What's Laplace with k = 0?
  - k is the strength of the prior
- Laplace for conditionals:
  - Smooth each condition:
  - Can be derived by dividing

$$P_{LAP,k}(x|y) = \frac{c(x,y)+k}{c(\cdot,y)+k|X|}$$

H H T

$$P_{LAP,0}(X) = \left\langle \frac{2}{3}, \frac{1}{3} \right\rangle$$

$\frac{1+1}{3+1\cdot 2}$

$$P_{LAP,1}(X) = \left\langle \frac{3}{5}, \frac{2}{5} \right\rangle$$

$$P_{LAP,100}(X) = \left\langle \frac{102}{203}, \frac{101}{203} \right\rangle$$

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Better: Linear Interpolation

$y=1$ $y=0$

- Linear interpolation for conditional likelihoods
  - **Idea**: the conditional probability of a feature x given a label y should be close to the marginal probability of x
  - **Example**: A rare word like "interpolation" should be similarly rare in both ham and spam (a priori)
  - **Procedure**: Collect relative frequency estimates of both conditional and marginal, then average

$\hat{P}(x|y), \hat{P}(x)$

$$P_{ML}(x|y) = \frac{\text{count}(x,y)}{\text{count}(\cdot,y)} \qquad P_{ML}(x) = \frac{\text{count}(x)}{\text{count}(\cdot)}$$

$$P_{LIN}(x|y) = (1-\alpha)P_{ML}(x|y) + (\alpha)P_{ML}(x)$$

$\alpha \in [0,1]$

  - **Effect**: Features have odds ratios closer to 1

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Real NB: Smoothing

- Odds ratios without smoothing:

$\frac{P(W|\text{ham})}{P(W|\text{spam})}$

south-west	:	inf
nation	:	inf
morally	:	inf
nicely	:	inf
extent	:	inf
...		

$\frac{P(W|\text{spam})}{P(W|\text{ham})}$

screens	:	inf
minute	:	inf
guaranteed	:	inf
$205.00	:	inf
delivery	:	inf
...		

Real NB: Smoothing

$\sum_{i=1} \alpha_i = 1$, $\alpha_i \geq 0$

- Odds ratios after smoothing:

$\frac{P(W|\text{ham})}{P(W|\text{spam})}$

helvetica	:	11.4
seems	:	10.8
group	:	10.2
ago	:	8.4
areas	:	8.3
...		

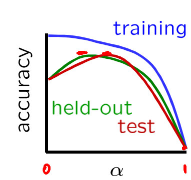
$\frac{P(W|\text{spam})}{P(W|\text{ham})}$

verdana	:	28.8
Credit	:	28.4
ORDER	:	27.2
<FONT>	:	26.9
money	:	26.5
...		

Do these make more sense?

Tuning on Held-Out Data

- Now we've got two kinds of unknowns
  - Parameters: $P(F_i|Y)$ and $P(Y)$
  - Hyperparameters, like the amount of smoothing to do: k, α
- Where to learn which unknowns
  - Learn parameters from training set
  - Can't tune hyperparameters on training data (why?)
  - For each possible value of the hyperparameters, train and test on the held-out data
  - Choose the best value and do a final test on the test data



Proportion of $P_{ML}(x)$ in $P(x|y)$

$\alpha=1$ $\alpha=0$

Baselines

- First task when classifying: get a baseline
  - Baselines are very simple “straw man” procedures
  - Help determine how hard the task is
  - Help know what a “good” accuracy is
- Weak baseline: most frequent label classifier
  - Gives all test instances whatever label was most common in the training set
  - E.g. for spam filtering, might label everything as spam
  - Accuracy might be very high if the problem is skewed
- When conducting real research, we usually use previous work as a (strong) baseline

Confidences from a Classifier

→ $P(\text{spam} \mid f_1 \ldots f_n) = .51$ $\quad P(f_1 \mid Y=1) = 0.1$

$0.5 \cdot (0.1)^k$

$P(Y=1) \cdot P(f_1 \mid Y=1) \cdot P(f_1' \mid Y=1) \ldots P(f_1''' \mid Y \ldots$

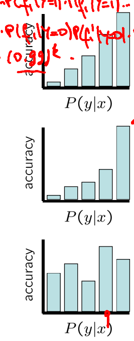
$P(Y=0) \cdot P(f_1 \mid Y=0) P(f_1' \mid Y=0) \cdot P(f_1'' \mid Y=0)$

$0.5 \cdot (0.99)^k$

- The confidence of a classifier:
  - Posterior of the most likely label

$$\text{confidence}(x) = \max_y P(y|x)$$

  - Represents how sure the classifier is of the classification
  - Any probabilistic model will have confidences
  - No guarantee confidence is correct
- Calibration
  - Strong calibration: confidence predicts accuracy rate
  - Weak calibration: higher confidences mean higher accuracy
  - What’s the value of calibration?



Naïve Bayes Summary

- Bayes rule lets us do diagnostic queries with causal probabilities
- The naïve Bayes assumption takes all features to be independent given the class label
- We can build classifiers out of a naïve Bayes model using training data
- Smoothing estimates is important in real systems
- Confidences are useful when the classifier is calibrated

What to Do About Errors

- Problem: there’s still spam in your inbox
- Need more features – words aren’t enough!
  - Have you emailed the sender before?
  - Have 1K other people just gotten the same email?
  - Is the sending information consistent?
  - Is the email in ALL CAPS?
  - Do inline URLs point where they say they point?
  - Does the email address you by (your) name?
- Naïve Bayes models can incorporate a variety of features, but tend to do best in homogeneous cases (e.g. all features are word occurrences)

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Generative vs. Discriminative

$\theta = \{P(Y), \{P(F_i \mid Y)\}\}$ $\quad \{(y^{(i)}, f_1^{(i)}, f_2^{(i)}, \ldots, f_n^{(i)})\}_{i=1}^{n}$ $\quad P_\theta(F, Y)$

- Generative classifiers:
  - E.g. naïve Bayes
  - A causal model with evidence variables
  - Query model for causes given evidence

① P_θ(training data) + $P_{reg.,\alpha,k}$

likelihood: $\prod_i P_\theta(y^{(i)}) \cdot \prod_{j=1}^{n} P(f_j^{(i)} \mid y^{(i)})$

→ #training data

→ $\theta = \arg\max_\theta L(\theta) + reg.(\theta)$

later: use $P_\theta(y \mid f_1 \ldots f_n)$

- Discriminative classifiers:
  - No causal model, no Bayes rule, often no probabilities at all!
  - Try to predict the label Y directly from X
  - Robust, accurate with varied features
  - Loosely: mistake driven rather than model driven

② find θ s.t. accuracy of prediction using $P_\theta(Y|X)$ is maximized training data

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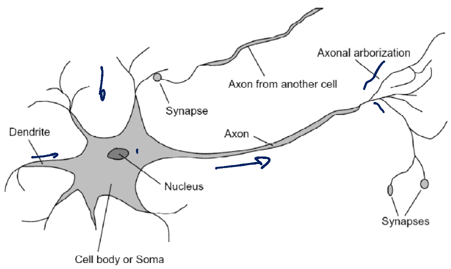
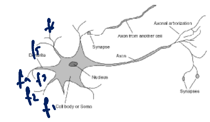
Some (Simplified) Biology

- Very loose inspiration: human neurons



Linear Classifiers

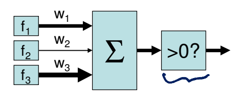
- Inputs are feature values
- Each feature has a weight
- Sum is the activation



$$\text{activation}_w(x) = \sum_i w_i \cdot f_i(x) = w \cdot f(x)$$

going out on the axon

- If the activation is:
  - Positive, output +1
  - Negative, output -1



Example: Spam

- Imagine 4 features (spam is "positive" class):
  - free (number of occurrences of "free")
  - money (occurrences of "money")
  - BIAS (intercept, always has value 1)

$w \cdot f(x)$

$\sum_i w_i \cdot f_i(x)$

x	$f(x)$	w	
"free money"	BIAS : 1 free : 1 money : 1 ...	BIAS : -3 free : 4 money : 2 ...	$(1)(-3)$ + $(1)(4)$ + $(1)(2)$ + ... = 3

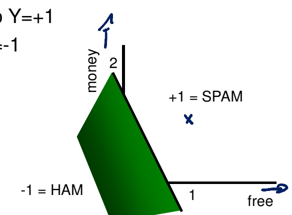
Binary Decision Rule

- In the space of feature vectors
  - Examples are points
  - Any weight vector is a hyperplane
  - One side corresponds to Y=+1
  - Other corresponds to Y=-1

$\boxed{w \cdot f > 0}$

w

```
BIAS  : -3
free  :  4
money :  2
...
```



$f \cdot w = 0$